The Equilibrium Structure of Cosmological Halos: From Dwarf Galaxies to X-ray Clusters

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Abstract*

A new model for the postcollapse equilibrium structure of virialized objects which condense out of the cosmological background universe is described and compared with observations and simulations of cosmological halos from dwarf galaxies to X-ray clusters. The model is based upon the assumption that virialized halos are isothermal, which leads to a prediction of a unique nonsingular isothermal sphere for the equilibrium structure, with a core radius which is approximately 1/30 times the size and a core density which is proportional to the mean background density at the epoch of collapse. These predicted nonsingular isothermal spheres are in good agreement with the observations of the internal structure of dark-matter-dominated halos from dwarf galaxies to X-ray clusters.

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Equilibrium structure of virialized halos

[Shapiro, Iliev, and Raga 1999, MNRAS, 307, 203 (Einstein-de Sitter case); Iliev and Shapiro 2000 (low-density, open and flat with cosmological constant)]

- Problem and Motivation
  - Question: What equilibrium structure forms when a density perturbation collapses out of the expanding background universe and virializes?
  - An analytical model for the structure (e.g. mass profile, temperature, velocity dispersion, radius) of virialized cosmological halos would be a valuable tool for the semi-analytical modeling of galaxy and cluster formation in a hierarchical clustering model like CDM.
    - Earlier work adopted crude approximations involving either uniform spheres or singular isothermal spheres which resulted from top-hat perturbation collapse
    - What is a more realistic outcome, even for the simple top-hat problem?
  - N-body simulations of CDM predict dark matter halo profiles with singular density profiles, but a finite density core is required to explain:
    - Dwarf galaxy rotation curves
    - Cluster mass profiles inferred from gravitational lensing
  - As a result, the cold, collisionless nature of CDM has recently been re-examined to allow for variations which affect the post-collapse equilibrium structure of halos.

- Suppose we ignore the details of this relaxation process and adopt the assumption that the final equilibrium is isothermal.
- Solve this problem and compare the result with dwarf galaxy rotation curves and X-ray cluster data.

- Model:
  - Top-hat density perturbation collapses and virializes
  - Virialization leads to a truncated isothermal sphere in hydrostatic equilibrium (TIS) $\Rightarrow$ solution of the Lane-Emden equation (modified for $\Lambda \neq 0$)
  - Total energy of top-hat is conserved thru collapse and virialization
  - Postcollapse temperature set by virial theorem (including effect of finite boundary pressure)

- Is the solution uniquely determined? – No, some additional information is required:
  1) Minimum-Energy Solution: Boundary pressure is that for which the conserved top-hat energy is the minimum possible for an isothermal sphere of fixed mass within a finite truncation radius.
  2) The Self-Similar Spherical Cosmological Infall Solution (Bertschinger 1985) confirms this choice if we identify the virialized object with the spherical region of post-shock gas and shell-crossing dark matter $\Rightarrow$ explains dynamical origin of boundary pressure adopted above as the result of thermalizing the energy of infall.

Summary of the TIS Solution

- Top-hat perturbation $\Rightarrow$ unique, **nonsingular** TIS (minimum-energy configuration)

  $\Rightarrow$ universal, self-similar density profile for the postcollapse equilibrium of cosmic structure

  - Unique scale and amplitude set by top-hat mass and collapse epoch
  - Same density profile for gas and dark matter (no bias)

I. Matter-Dominated Case (see Table 1 and Fig. 1)

  - Finite core size: $r_0 = 0.034 \times$ radius r_t
  - Central density: $\rho_0 = 514 \times$ surface density ρ_t
  - $T = 2.16\ T_{\text{uniform sphere}} = 0.72\ T_{\text{singular isothermal sphere}}$
  - At intermediate radii, ρ drops faster than r^{-2}

II. Flat, $\Lambda \neq 0$ Case

  - Profile varies with epoch of collapse, approaching case I above for early collapse.
    For example: for $\Omega_0 = 1 - \lambda_0 = 0.3$ for $z_{coll} = (0; 0.5; 1)$:
    * $r_t/r_0 = (30.04; 29.68; 29.54)$
    * $\rho_0/\rho_t = (529.9; 520.8; 517.2)$
    * $T/T_{\text{uniform sphere}} = (2.188; 2.170; 2.163)$

Table 1: The Postcollapse Virial Equilibrium Resulting from the Collapse of Top-Hat Density Perturbations: Einstein-de Sitter Universe

	Uniform Sphere	Singular Isothermal Sphere	Our Solution*
$\eta = \frac{r_t}{r_m}$.....	0.5	0.417	0.554
$\frac{k_B T_{\rm vir}}{\left(\frac{2}{5}\frac{GMm}{r_{\rm vir}}\right)}$...	1	3	2.16
$\frac{\rho_0}{\rho_t}$...........	1	∞	514
$\frac{\langle\rho\rangle}{\rho_t}$...........	1	3	3.73
$\frac{r_t}{r_0}$.............	– NA –	∞	29.4
$\frac{\langle\rho\rangle}{\rho_b(t_{\rm coll})}$.....	$18\pi^2$ ≈ 178	$18\pi^2\left(\frac{6}{5}\right)^3 \approx \pi^5$ ≈ 307	130.5

* Our solution = minimum-energy, truncated, nonsingular, isothermal sphere

Note: $\rho_b \equiv$ cosmic mean matter density

Density Profile of Halo which Forms from Top-Hat Perturbation Collapse and Virialization

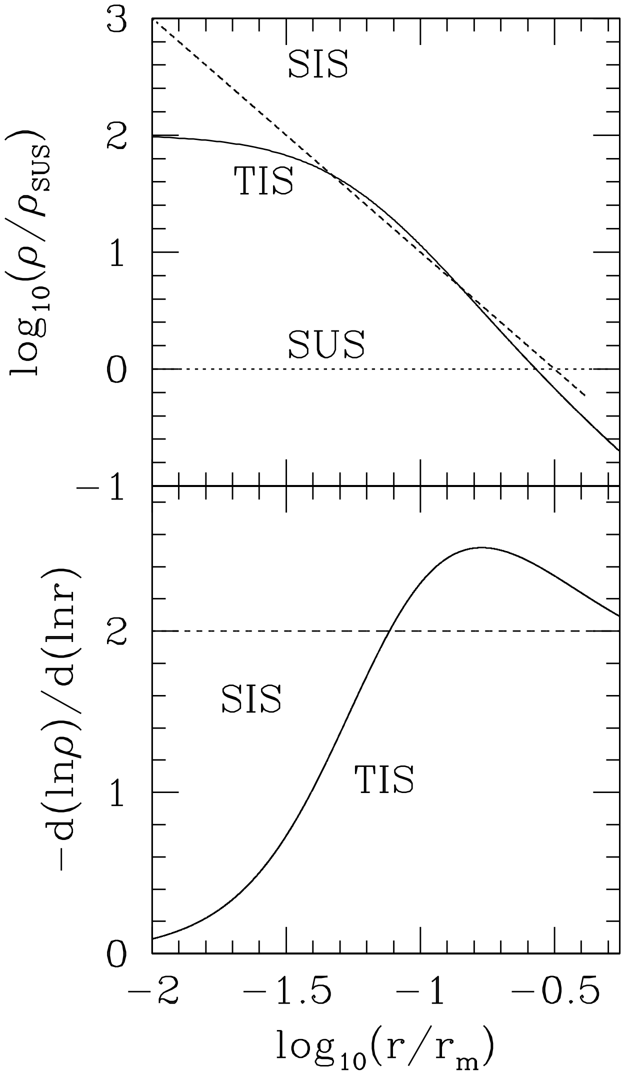


Fig.1: Density profile of truncated isothermal sphere which forms from the virialization of a top-hat density perturbation in a matter-dominated universe. Radius r is in units of r_m - the top-hat radius at maximum expansion, while density ρ is in terms of the density ρ_{SUS} of the standard uniform sphere approximation for the virialized, post-collapse top-hat. (TIS = our solution, SUS = uniform sphere, SIS = singular isothermal sphere). Bottom panel shows logarithmic slope of density profile.

Direct Comparison with NFW Profile

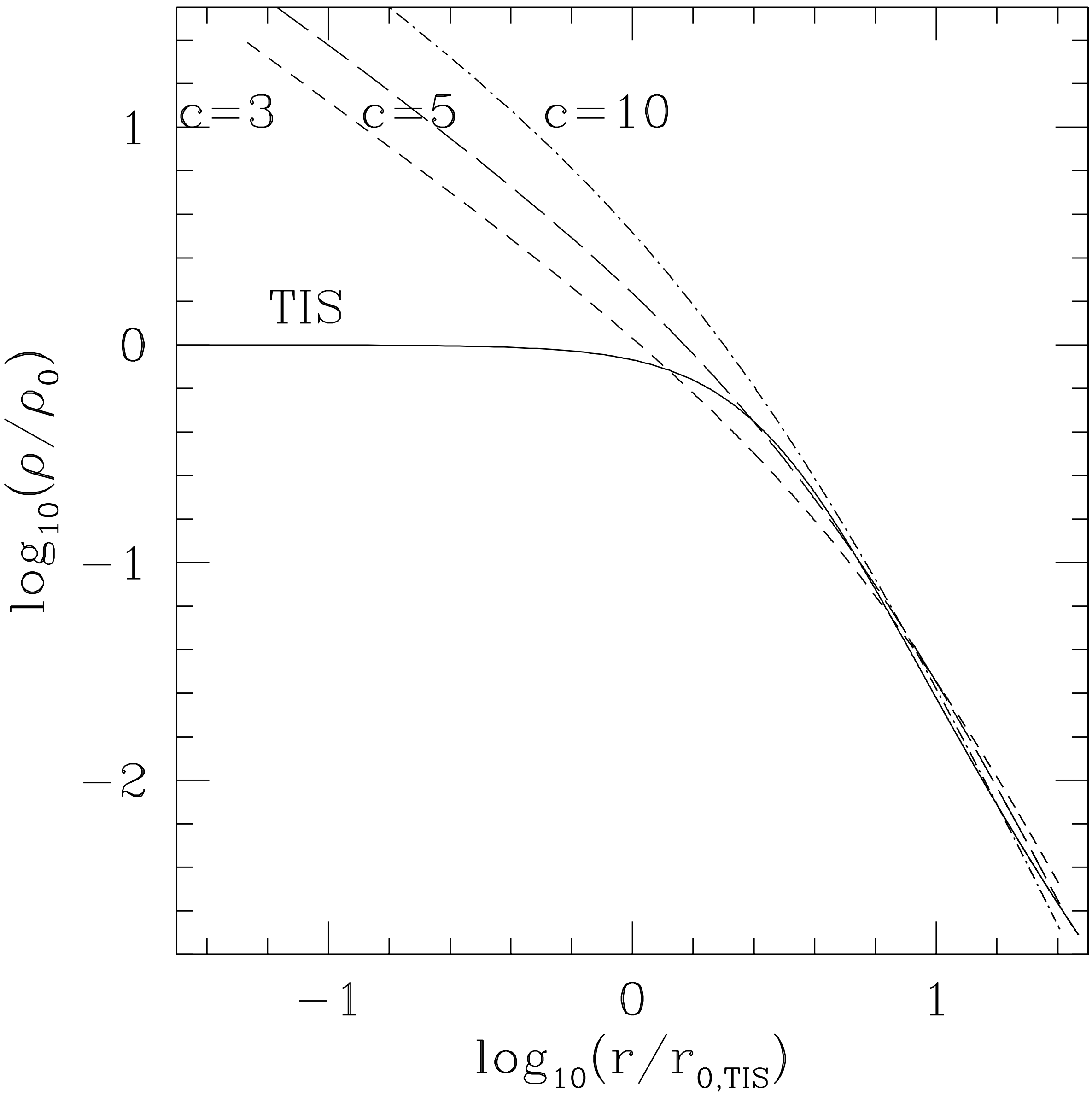


Fig. 2: Continuous line = TIS profile; Dashed lines = "NFW" = Navarro, Frenk, and White (1996, 1997) profile:

$$\rho(r) = \frac{\delta_c \rho_{b0}}{cx(cx+1)^2}, \qquad x = \frac{r}{r_{200}}$$

Range of c appropriate for X-ray clusters to early forming dwarf galaxies.

Dwarf Galaxy Rotation Curves

Q: How well does our TIS profile match the observed mass profiles of dark-matter-dominated dwarf galaxies? The observed rotation curves of dwarf galaxies can be fit according to the following density profile with a finite density core (Burkert 1995):

$$\rho(r) = \frac{\rho_{0,Burkert}}{(r/r_c + 1)(r^2/r_c^2 + 1)}$$

A: The TIS profile gives a nearly perfect fit to the Burkert profile. (see Fig. 3)

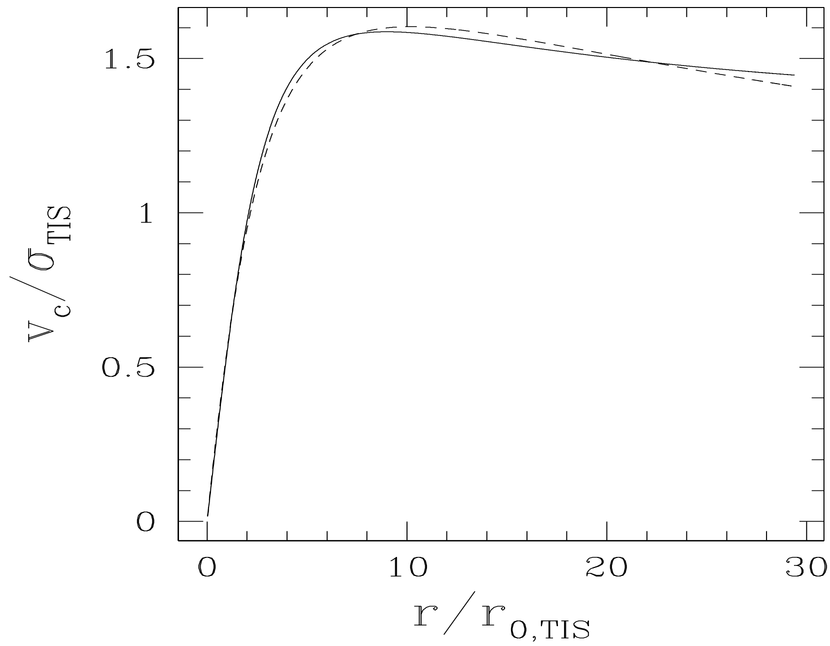


Fig. 3: Rotation Curve Fit. Best fit parameters:

$$\frac{\rho_{0,Burkert}}{\rho_{0,TIS}} = 1.216, \; \frac{r_c}{r_{0,TIS}} = 3.134$$

Solid line = Best fit TIS; Dashed line = Burkert profile, where $\sigma_{TIS}^2 = \langle v^2 \rangle /3 = k_B T/m$.

Q: How well does this best fit TIS profile predict the r_{max} and v_{max}?

A: $\frac{r_{max,Burkert}}{r_{max,TIS}} = 1.13, \; \frac{v_{max,Burkert}}{v_{max,TIS}} = 1.01$

(i.e. excellent agreement)

The $v_{max} - r_{max}$ relation for dwarf and LSB galaxies.

Q: Can the TIS halo model explain the observed correlation of v_{max} and r_{max} for dwarf spiral and LSB galaxies?

A: Yes, when the TIS halo model is combined with the Press-Schechter model which predicts the typical collapse epoch for objects of a given mass (i.e. the mass of the 1σ-fluctuations vs. z_{coll}). (See Fig. 4) For the three untilted CDM models plotted, a cluster normalized Einstein-de Sitter model, and COBE-normalized low-density models ($\Omega_0 = 0.3$ and $\lambda_0 = 0$ or 0.7), only the flat models yield a reasonable agreement with the observed $v_{max} - r_{max}$ relation.

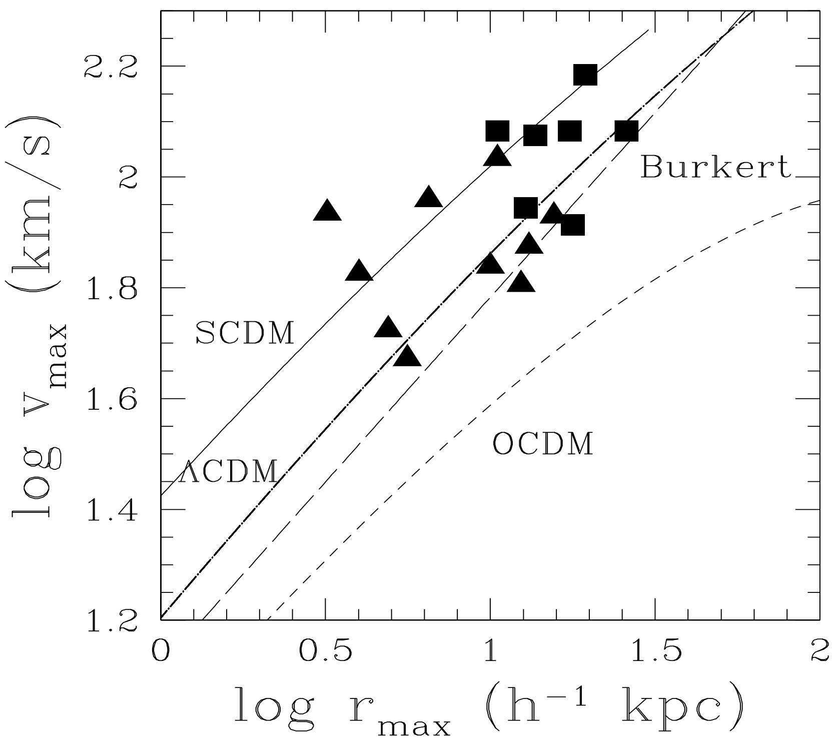
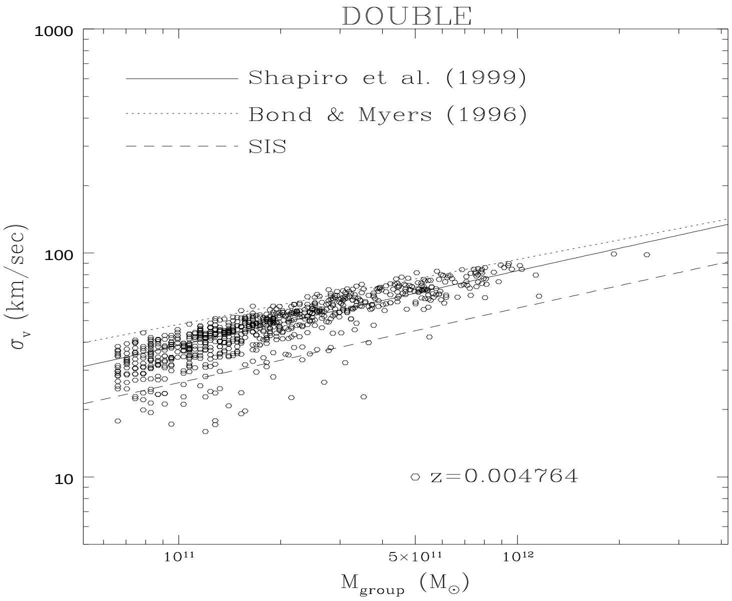


Fig. 4: Dwarf galaxies (triangles) and LSB galaxies (squares) from Kravtsov et al. (1998); Burkert: fit to data (Mori & Burkert 2000); SCDM: $\Omega_0 = 1$, $\lambda_0 = 0$, $\sigma_{8h^{-1}} = 0.5$ (cluster normalized); OCDM: $\Omega_0 = 0.3$, $\lambda_0 = 0$ (COBE normalized); ΛCDM: $\Omega_0 = 0.3$, $\lambda_0 = 0.7$, (COBE normalized); $h = 0.65$ for all.

Galaxy Halo $M - \sigma_v$ Relation

Q: How well does our TIS halo model scaling relation predict the velocity dispersion of galactic halos which form in the CDM model according to N-body simulations?

A: Antonuccio-Delogu, Becciani, & Pagliaro (1999) used an N-body treecode at high-res (256^3 particles) to simulate galactic halos in the region of a single and a double cluster. The agreement with the TIS model is good.



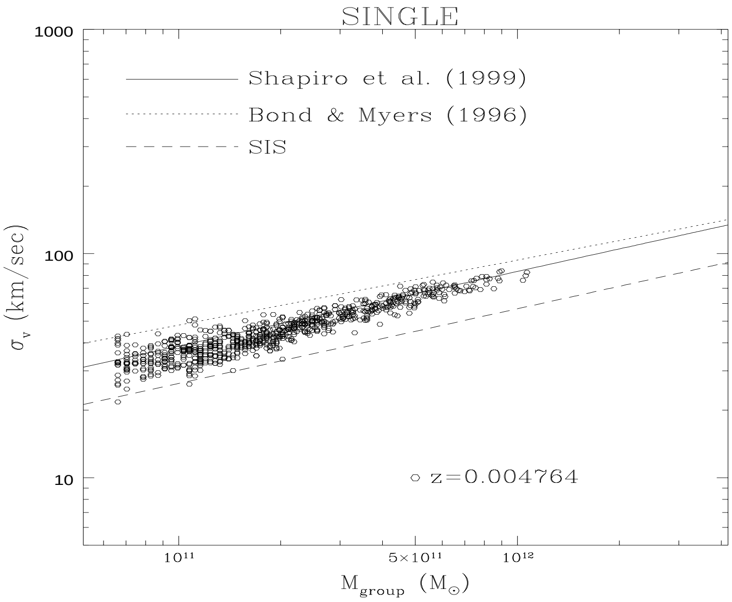


Fig. 6: Velocity dispersion vs. mass for galactic haloes in cluster regions: (upper panel) double cluster, (lower panel) single cluster.

X-Ray Cluster Scaling Relations

Q: How well does the TIS halo model predict the internal structure of X-ray clusters found by gas-dynamical/N-body simulations of X-ray cluster formation in the CDM model?

A: As shown below and in Fig. 5, our TIS model predictions agree astonishingly well with the mass-temperature and the radius-temperature virial relations and integrated mass profiles derived from numerical simulations by Evrard, Metzler and Navarro (1996). Apparently, these simulation results are not sensitive to the discrepancy between our prediction of finite density core and the N-body predictions of a density cusp for clusters in CDM.

- Mass Profile – Temperature Relation

$$r_X \equiv r_{10}(X)\left(\frac{T}{10\,\mathrm{keV}}\right)^{1/2}; \qquad X \equiv \frac{\langle\rho(r)\rangle}{\rho_b}$$

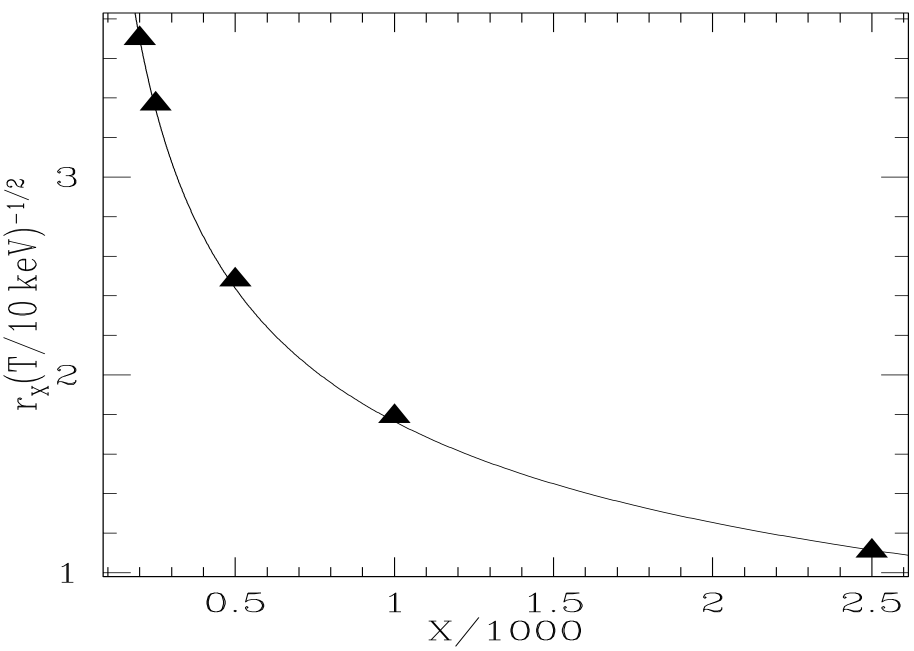


Fig.5 (triangles) fit to CDM simulation results by Evrard, Metzler and Navarro (1996); (continuous line) TIS prediction.

- Mass-Temperature and Radius-Temperature Virial Relations
  - Evrard, Metzler and Navarro (1996)

$$
\begin{aligned}
M_{500} &= (1.11 \pm 0.16) \times 10^{15} \left(\frac{T}{10\,\mathrm{keV}}\right)^{3/2} h^{-1}\, M_\odot, \\
r_{500} &= (1.24 \pm 0.09) \left(\frac{T}{10\,\mathrm{keV}}\right)^{1/2} h^{-1}\,\mathrm{Mpc}. \\
M_{200} &= 1.45 \times 10^{15} \left(\frac{T}{10\,\mathrm{keV}}\right)^{3/2} h^{-1}\, M_\odot, \\
r_{200} &= 1.85 \left(\frac{T}{10\,\mathrm{keV}}\right)^{1/2} h^{-1}\,\mathrm{Mpc}.
\end{aligned}
$$

  - Our solution

$$
\begin{aligned}
M_{500} &= 1.11 \times 10^{15} \left(\frac{T}{10\,\mathrm{keV}}\right)^{3/2} h^{-1}\, M_\odot, \\
r_{500} &= 1.24 \left(\frac{T}{10\,\mathrm{keV}}\right)^{1/2} h^{-1}\mathrm{Mpc}\,. \\
M_{200} &= 1.55 \times 10^{15} \left(\frac{T}{10\,\mathrm{keV}}\right)^{3/2} h^{-1}\, M_\odot, \\
r_{200} &= 1.88 \left(\frac{T}{10\,\mathrm{keV}}\right)^{1/2} h^{-1}\,\mathrm{Mpc}.
\end{aligned}
$$

β-fits to X-ray Cluster Brightness and Density Profiles

$$\rho_{gas} = \frac{\rho_0}{\left(1 + r^2/r_c^2\right)^{3\beta/2}}, \quad I = \frac{I_0}{\left(1 + \theta^2/\theta_c^2\right)^{3\beta - 1/2}}$$

Q: How well does the TIS model for the internal structure of X-ray clusters predict the observed and simulated X-ray brightness profile of clusters?

A: It predicts gas density profiles and brightness profiles which are well-fit by a $\beta-$profile, with β-values for the TIS β-fit which are close to those of simulated clusters in the CDM model, but somewhat larger than the conventional observational result that $\beta \approx 2/3$. However, recent X-ray results suggest that the true β-values are larger than 2/3 when measurements at larger radii are used and when central cooling flows are excluded from the fit.

Brightness profile observations	β
Jones and Foreman (1999)	0.4-0.8, ave. 0.6
Jones and Foreman (1992)	$\sim 2/3$
Balland and Blanchard (1997)	0.57 (Perseus) 0.75 (Coma)
Durret et al. (2000)	0.53 (incl. cooling flow) 0.82 (excl. cooling flow)
Vikhlinin, Forman, & Jones (1999) (fit by Henry 2000)	0.7-0.8
TIS β-fit ($r_c/r_{0,TIS} = 2.639$)	0.904

Gas density profile simulations	β
Metzler and Evrard (1997)	0.826 (DM) 0.870 (gas)
Eke, Navarro, and Frenk (1998)	0.82
Lewis et al. (1999) (adiabatic)	~ 1
Takizawa and Mineshige (1998)	~ 0.9
Navarro, Frenk, and White (1995)	0.8
TIS β-fit ($r_c/r_{0,TIS} = 2.416$)	0.846

X-ray Cluster Gas Entropy

Q: Can the TIS model for the internal structure of X-ray clusters explain the observed correlation between the gas entropy near the cluster center and cluster virial temperature?

A: Yes, but only for high-T clusters (i.e. $T >$ few keV) for which energy release feedback effects were probably not big enough to alter the entropy of the equilibrium halo. (See Fig. 7)

$$S = T/n_e^{2/3}, \text{at } r = 0.1 r_{vir}$$

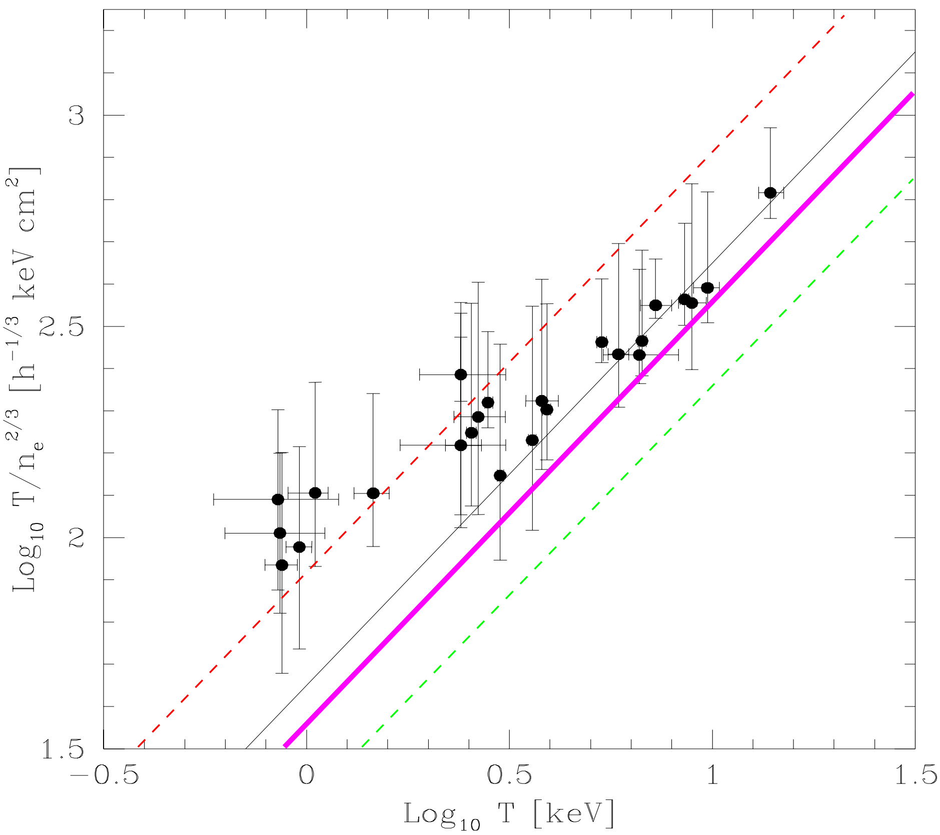


Fig. 7: Cluster entropy vs. temperature. Data: Ponman, Cannon, and Navarro (1999) Nature, 397; Error bars: T – 90% confidence level, entropy – span of variation from $r = 0.05 r_{vir}$ to $r = 0.2 r_{vir}$. Our solution: thick line = S at $r = 0.1 r_{vir}$, dashed lines = S at $r = 0.05 r_{vir}$ (lower), and $0.2 r_{vir}$ (upper).

Q: Can the TIS halo model explain the mass profile with a finite density core measured by Tyson, Kochanski, and Dell'Antonio (1998) for cluster CL 0024+1654 at $z = 0.39$ using the strong gravitational lensing of background galaxies by the cluster to infer the cluster mass distribution?

A: Yes, the TIS model not only provides a good fit to the shape of the projected surface mass density distribution of this cluster (see Fig. 8) within the arcs, but when we match the central value as well as the shape, our model predicts the overall mass, and a cluster velocity dispersion in close agreement with the value $\sigma_v = 1200$ km/s measured by Dressler and Gunn (1992).

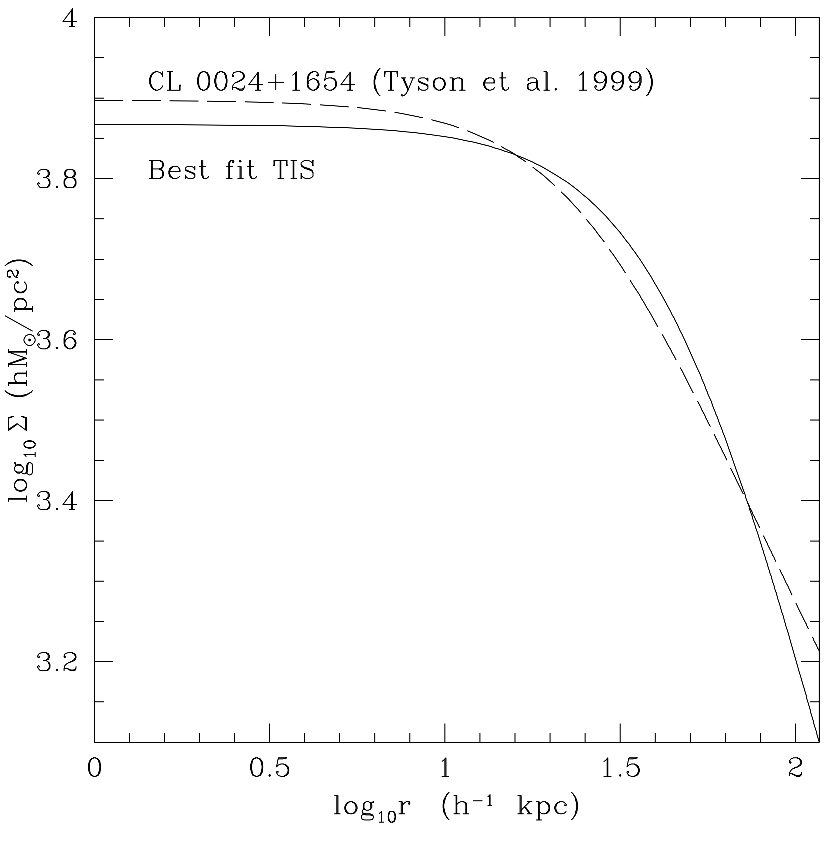


Fig. 8: Projected surface density of cluster CL 0024+1654 inferred from lensing measurements, together with the best fit TIS model.

Summary

- TIS profile fits dwarf galaxy rotation curves; combined with Press-Schechter formalism matches results for observed v_{max}–r_{max} relation for dwarf galaxies

- Predicted mass-velocity dispersion relation agrees with high resolution N-body simulations of galactic halo formation by Antonuccio-Delogu et al. (1999)

- Predicted mass-radius-temperature scaling relations match simulation results from X-ray clusters in CDM model

  - Our solution **derives** empirical fitting formulae of Evrard, Metzler and Navarro (1996)

  - Agrees well with X-ray cluster observations at $z = 0$

- Fits high temperature X-ray cluster entropy data

- X-ray brightness profile is predicted to match β-fit with $\beta \approx 0.9$, larger than typically observed, but similar to results of gas-dynamical/N-body simulations of X-ray clusters in CDM model

- Fits the cluster mass profile with finite core derived from strong gravitational lensing data of Tyson et al. (1999) on CL 0024+1654

- Predicted mass profile is close to NFW profile for low values of concentration parameter, outside the core

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